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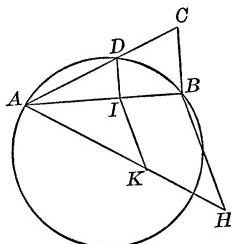
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SOLUTION BY EMMA M. GIBSON, Drury College.

The ruler and compasses are sufficient to construct a triangle whose hypotenuse shall be divided in any ratio as  $m : n$ , according to the conditions of the problem.



For let  $AB$  be the given chord. From  $A$  draw any line as  $AH$  and from  $A$  lay off  $AK = m$  and  $KH = n$ . Then connect  $B$  and  $H$  and draw  $KI$  parallel to  $HB$  and intersecting  $AB$  in  $I$ . At  $I$  erect a perpendicular to  $AB$  intersecting the circle in  $D$ . Join  $A$  and  $D$  by a straight line and produce it until it meets a perpendicular erected at  $B$  in  $C$ . Then the right triangle  $ABC$  has its hypotenuse,  $AC$ , divided at  $D$  in the ratio of  $m : n$ .

For in the similar triangles  $AIK$  and  $ABH$ ,  $AI : IB = m : n$  and in the similar triangles  $ABC$  and  $AID$ ,  $AI : IB = AD : DC$ . Hence,  $AD : DC = m : n$ .

When  $m = n$ ,  $D$  bisects the hypotenuse.

Also solved by CLIFFORD N. MILLS, F. M. MORGAN, C. HORNUNG, HORACE OLSON, and S. W. REAVES.

**429. Proposed by JOHN A. BIGBEE, Little Rock, Ark.**

In the trihedral angle  $V-ABC$ , the face angle  $AVB$  is bisected by the straight line  $VD$ . Is it true that the angle  $DVC$  is less than, equal to, or greater than, half the sum of the angles  $AVC$  and  $BVC$ , according as  $\angle CVD$  is less than, equal to, or greater than  $90^\circ$ ?

SOLUTION BY A. M. HARDING, University of Arkansas.

(1)  $\angle CVD < 90^\circ$ . (Fig. 1.) Take  $VA = VB$ . Join  $A$  and  $B$ . Let this line cut the bisector of  $\angle AVB$  at  $D$ . Through  $D$  pass a plane perpendicular to

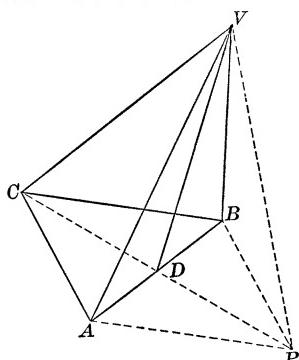


FIG. 1.

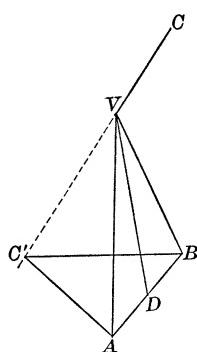


FIG. 2.

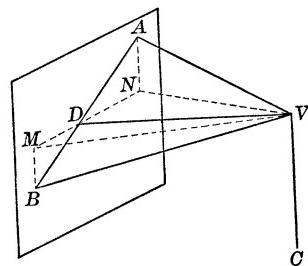


FIG. 3.

$VD$  cutting  $VC$  at  $C$ . Produce  $CD$  to  $P$  making  $DC = DP$ . Draw the lines  $AP$ ,  $BP$ , and  $VP$ . From this construction it easily follows that  $\angle BVC = \angle PVA$  and  $\angle CVP = 2\angle CVD$ . But  $\angle CVP < \angle AVC + \angle PVA$ . Hence  $\angle CVD < \frac{1}{2}(\angle AVC + \angle BVC)$ .

(2)  $\angle CVD > 90^\circ$ . (Fig. 2.) Produce  $CV$  through  $V$  and draw plane  $ABC'$  as in (1). Then  $2\angle C'VD < \angle AVC' + \angle BVC'$  by (1). Hence  $2(180^\circ - \angle C'VD) > 180^\circ - \angle AVC' + 180^\circ - \angle BVC'$  or  $\angle CVD > \frac{1}{2}(\angle AVC + \angle BVC)$ .

(3)  $\angle CVD = 90^\circ$ . (Fig. 3.) In this case the plane through  $AB$  is parallel to  $VC$ . Draw a plane  $MNV$  through  $VD$  perpendicular to  $VC$  and cut by planes  $BVC$  and  $AVC$  in the lines  $MV$  and  $NV$  respectively. It can be easily shown that  $\angle MVB = \angle NVA$ . Hence

$$\angle AVC + \angle BVC = 180^\circ$$

and

$$\angle CVD = \frac{1}{2}(\angle AVC + \angle BVC).$$

Also solved by B. LIBBY, F. M. MORGAN, and GEO. W. HARTWELL.

#### CALCULUS.

##### 338. Proposed by RICHARD LOCHNER, Philadelphia, Pa.

An elliptical field has a major axis of 100 feet and a minor axis of 10 feet. A cow is tethered at the end of the major axis and another at the end of the minor axis. If each cow can graze over half the field, how long is the rope of each? What is the area of the portion over which the cows can graze in common?

SOLUTION BY B. F. FINKEL, Drury College.

The central equation of the elliptic field is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Let  $r_1$  be the length of rope by which one cow is tethered at the point  $A_1$ , the right-hand extremity of the major axis. The equation of the circle over which this cow can browse is  $(x - a)^2 + y^2 = r^2$ . The coördinates of the point of intersection,  $P_1$ , of this circle with the ellipse are

$$(x_1, y_1) = \left( a \frac{[a^2 - \sqrt{b^4 + r^2(a^2 - b^2)}]}{(a^2 - b^2)}, \sqrt{2a^4 \frac{\sqrt{b^4 + r^2(a^2 - b^2)} - b^2(a^2 - b^2)r^2 - a^2(a^4 + b^4)}{(a^2 - b^2)}} \right),$$

The area common to the ellipse and the circle is

$$\begin{aligned} \text{area} &= 2 \int_0^{y_1} \int_{a - \sqrt{r^2 - y^2}}^{(a/b)\sqrt{b^2 - y^2}} dx dy \\ &= 2 \left[ \frac{a}{b} \frac{y_1}{2} \sqrt{b^2 - y_1^2} + \frac{b^2}{2} \sin^{-1} \frac{y'}{b} - ay' + \frac{y'}{2} \sqrt{r^2 - y_1^2} + \frac{r^2}{2} \sin^{-1} \frac{y_1}{r} \right], \end{aligned}$$

which, by the conditions of the problem, =  $\frac{1}{2}\pi ab$ .